A STOCHASTIC MODEL FOR DETERMINING INSPECTION INTERVALS FOR LARGE MARINE VESSELS

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ABSTRACT

This paper presents a stochastic model that accounts for failure probability as a function of inspection frequency and effectiveness. A total ownership cost (TOC) model is presented. The model is then applied to a cargo vessel. Stress-strength relationships predict the failure likelihood of details or critical points within the structure. These individual points are combinatorially combined into a single higher level model. An inspection plan together with the effectiveness of inspection with and without NDE is then used to determine the overall failure probability as a function of time and inspection intervals. Inputs into the model include the effectiveness of the NDE, the strength degradation and the stress distribution as a function of time, the cost of the NDE method, the effectiveness of the NDE method, the effectiveness of the inspection without NDE, and the cost of failure. Cost benefits can then be determined on the basis of a decreased failure likelihood with better detection methods or, alternatively, increased inspection intervals given a constant acceptable failure probability. Presentation of a prototype of this methodology implemented in MATLAB and using a military cargo ship as an example is presented.

Keywords: Cost Model, inspection intervals, cargo vessel, reliability, total ownership cost

1. INTRODUCTION

Risk-based methods for inspection and maintenance can significantly reduce lifecycle cost by basing inspection and repair intervals on the risk of incurring damage rather than on arbitrary periods. Not only can the methods reduce inspection cost and downtime, they can actually increase ship reliability and safety by defining explicit failure probabilities for all important components and functions. In this effort, we developed a prototype for a Risk Based Maintenance Strategy Tool (RBMST) to support a risk-based maintenance approach for large oceangoing vessels. The innovations of our approach include

- the use of a probabilistic fracture mechanics model as opposed to the traditional SN curves for structural details
- the integration of structural details into a complete ship model using a combinatorial probabilistic approach
- the development of an economic model that incorporates inspection and maintenance costs and the risk of vessel loss to allow for a cost-based assessment of maintenance strategy alternatives.

2. COST MODEL

The cost justification for embedding inspection and monitoring devices into civil structures is the reduction in the total ownership cost (TOC). The components of TOC that can be most affected by such devices are the cost of periodic inspections, the frequency of such inspections, the restoration cost saved by earlier detection, and the reduction in failure probability given constraints in the cost of frequency of such inspection. In some cases, the structure may be out of service during the time of the inspection, in which case the cost of the inspection is also affected by the capability of the inspection methods and internal devices used in the structure.

An economically optimal inspection and maintenance plan for will minimize the sum of maintenance, outage time, and risk. That is, to minimize cost where

$$TOC = \sum_{i=1}^{N} PV\left[\sum_{j=1}^{M} p_j q_j + d_j r_j\right] + \sum_{k=1}^{P} C_k \int_0^T f_k(t)dt \qquad \textbf{Equation 1}$$

Where

TOC is the total ownership cost
PV is the present value function
i=repair/inspection occurrence index
j=inspection type index
k=consequence type index
N=total number of inspections
M=total number of inspection and repair types
P=total number of consequence types
T= life time
p=unit cost of inspection method
q=quantity of inspection required
d=unit cost (rate) of repair
r=amount of repair required as a result of inspection
C=consequence value
f=failure probability

Equation 1 predicts total ownership cost as a function of the cost of inspections over the life time of the structure (first term inside the PV function, the cost of repairs as a result of the inspection, second term), and the risk of loss (consequences times probability, indicated in the integral sign). The relationship between these effectiveness parameters and the total cost is shown in Figure 1.

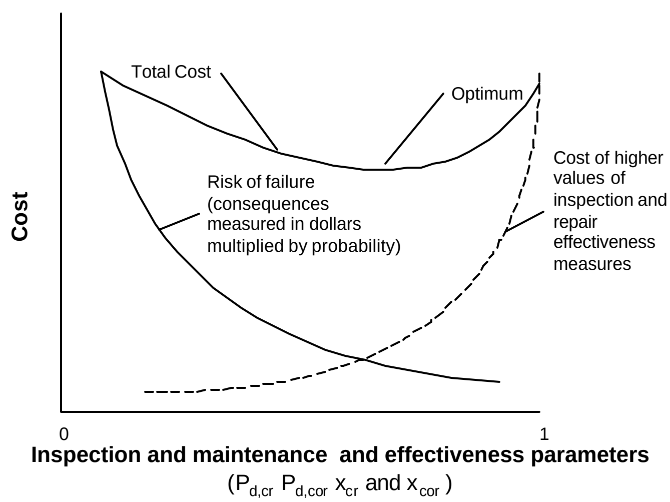

Figure 1 . Cost Aspects of Risk-Based Maintenance and Inspection Strategy

3. APPLICATION TO A CARGO VESSEL

Cargo vessels are subject to cyclic stresses due to waves as they travel across oceans. Their structures are subjected to cracking due to these stresses as well as corrosion due to aging. Ships fail when the stresses on structural elements (called details) of their structures exceed their strength. Over time, the strength of the details decrease due to corrosion and cracking. However, the strength can be renewed by means of repair. The likelihood of renewal is in turn related to the likelihood of detection, which is how the cost-effectiveness of NDE can be assessed. In this section, we develop a model for the reliability of the vessel based on (a) the likelihood of stress exceeding structural element strength and (b) the likelihood of the renewal of strength due to inspection and repair.

The reliability of the vessel (i.e., the probability of a failure over a unit time), can be expressed as the product of four lower level reliabilities reflecting whether or not cracking, corrosion, or both has occurred

$$R_{tot} = 1-\{[1-R_1] + [1-R_2] + [1-R_3] + [1-R_4]\} \qquad \textbf{Equation 2}$$

where each of these four reliabilities is shown in table 2

Table 1 Component Conditional Reliability Expressions

R_1	Conditional reliability without cracking and corrosion at time T	$R_1(T) = [1-F_{T_1}(T)][1-F_{T_o}(T)]R_{b,i,o}(T)$ **Equation 3**
R_2	Conditional reliability at Time T with corrosion starting at time t_o but no cracking	$R_2(T) = [1-F_{T_1}(T)]\int_0^T R_{b,i,o}(t_o)R_{b,i;a,o}(T-t_o)f_{T_o}(t_o)dt_o$ **Equation 4**
R_3	Conditional reliability at time T with cracks initiating at time t_i but no corrosion	$R_3(T) = \int_0^T R_{b,i,o}(t_i)R_{a,i;b,o}(T-t_i)f_{T_i}(t_i)dt_i[1-F_{T_o}(T)]$ **Equation 5**
R_4	Conditional reliability at time T with both cracks and corrosion starting at times t_i and t_o respectively	$R_4(T) = \int_0^T R_{b,i,o}(t_i)R_{a,i,o}(T-t_i)f_{T_i}(t_i)dt_i \int_0^T R_{b,i,o}(t_o)R_{a,i,o}(T-t_o)f_{T_o}(t_o)dt_o$ **Equation 6**
Note that these are *conditional* probabilities that, because of the integration, account for the likelihood that the part is in the corrosion or cracking state as well as the likelihood that it will fail given that it is in that state		

T is the time at which the reliability is evaluated, T_i is the time to crack initiation, T_o is the anticorrosion coating life (i.e., the time to initiation of corrosion), F_{Ti} (t) and F_{To} (t) are the cumulative probabilities for crack initiation and coating failure (respectively) at time t; f_{To} (t) and f_{Ti} (t) are the failure density functions (instantaneous failure probability functions) for crack initiation and coating failure (respectively) at time t; $R_{b,i,o}$ is the reliability before cracking failure, $R_{a,i;\ b,o}$ is the reliability after crack initiation and before coating failure; $R_{b,o;\ a,i}$ is the reliability after crack coating failure and before crack initiation, and $R_{a,i,o}$ is the reliability after both crack initiation and coating failure. The reliabilities $R_{b,i,o}$, $R_{a,i;\ b,o}$, $R_{b,o;\ a,i}$, and $R_{a,i,o}$, are in turn related to the likelihood of a stress being greater than the yield strength of the structural elements under the conditions of no cracking and no corrosion, cracking without corrosion, corrosion without cracking, or both cracking and corrosion respectively.

The risk can also be calculated on the basis of the multiplication of the failure probability and the consequence, that is

$$Risk = (1-R_{tot})*C \qquad \textbf{Equation 5a}$$

Both reliability and risk can be calculated on an individual element basis, in which case we would speak of $R_{tot,i}$ and $Risk_i$ to represent a specific element i.

A probabilistic fracture mechanics approach to deal with the sizes of the defects allows an evaluation of the crack growth under cyclic loading. Several methodologies for determining such cracking are available including a model by de Souza and Ayoub [Souza, 2000], Soares and Garbatov [Soares, 1999], and Casello and Rizutto [Casello, 1998].

For example, the $R_{b,i,o}$ term, the reliability before crack initiation and coating failure, is given by [Soares, 1999] as

$$R_{b,i,o} = \exp\left\{-\exp\left[\frac{s_s W_y(0) - M_{sw}}{a_{sw}}\right] n_o t\right\} \qquad \textbf{Equation 7}$$

where t is less than the time to first crack initiation, σ_s is the allowable stress, M_{sw} is the vertical bending moment of the hull member in still water, W_y is the vertical midship section modulus, a_{sw} is a constant, ν_o is the frequency of stresses that exceed σ_s (due to wave action, determined by wave distribution tables produced by organizations such as the American Bureau of Shipping or the British Martime Technologies [Feltham, 1986]. The other reliability terms, $R_{a,i;\ b,o}$, $R_{b,o;\ a,i}$, and $R_{a,i,o}$, (in the presence of cracking, corrosion, or both) are in turn related to the presence and depth of cracks (i.e., the cross section of the structural element that has not cracked) and corrosion (i.e., the cross section of the element that has not been corroded).

$R_{a,i,\ b,o}$, is determined using the limit state function [de Souza 2000] as a probablistic method of determining reliability based on the definition of a performance function related to Miner's rule. The limit state function is defined as

$$g_{\Delta K}(\underline{X}) = \int_{a_i}^{a_f} \frac{da}{[f(a)]^m \left(\sqrt{pa}\right)^m} - C\sum_{j=1}^{k} p_j \left[\sum_{i=1}^{n} \left(\Delta S_i\right)^m\right]_j \qquad \textbf{Equation 8}$$

where $g_{\Delta K}(\underline{X})$ is a vector of random variables including a_i, initial crack size present in the structure, a_f, the maximum crack size permissible in the structure, C, the is the Paris Erdogan equation [Paris 1963] constant for the material, $\Sigma\Delta S$ is the cumulative stress data acting on the structure over a given time due to loading condition j (a combination of the wave type and vessel cargo) and p_j is the probability of that loading condition. The failure condition for this equation is $g_{\Delta K}(\underline{X}) < 0$. The function f(a) represents the geometry of the elements, and, in accordance with the methodology defined by [de Souza 2000], is set such that the results of the limit equations match those of the the S-N equations (to make use of the large amount of experimental data).

Evaluation of the $R_{b,\ i,a\ o}$ term will use the rate of corrosion degradation on mild steel as a function of temperature and other conditions that has been described by Melchers [Melchers, 1999]. The integration of these relations into the equations shown in Table 1 has been done Soares and Garbatov [Soares, 1999] but are not discussed in further detail because of space limitations.

In the absence of inspection, maintenance and repair, the reliability of the structural elements will degrade over time. However, if they are detected and repaired, then the reliability will increase (although not necessarily back to the original reliability of the hull). The likelihood of crack detection within a given structural element depends on the technique and the minimum crack size detectable by each technique and is determined by the following relation [Soares, 1999]

$$P_{d,cr,i}(t) = 1 - \exp\left[-\frac{E[a_i(t)] - a_{d,o}}{l_d}\right] \qquad \textbf{Equation 9}$$

where $P_{d,cr,i}(t)$ is the probability of finding a crack within a specific structural element, $E[a_i(t)]$ is the average crack size at time t, $a_{d,o}$ is the minimum detectable crack size, λ_d is an empirically determined constant, and $C_{d,cr,i}$ is the consequences of the failure of that critical element. In Equation 8 it is assumed that the average crack size is larger than the minimum detectable crack size. Equation 8 predicts the detection likelihood for a particular critical element. Inaccessible locations will have a lower detection likelihood (or be more costly to inspect for a given detection likelihood), and the choice of inspection techniques (and the amount of time and resources devoted to inspection) will impact the $a_{d,o}$ and λ_d parameters.

The decision of which inspection techniques to use on which critical elements in turn determines the reliability of that critical element. The effectiveness of the inspection and repair method, when combined the consequence of failure of that structural element, is the risk of that element. The RBMST will allow the user to evaluate the impact of varying such parameters as part of the risk-based development of maintenance strategies.

At the ship level, the average probability of detection of cracked elements is given by

$$P_{d,cr} = \frac{\sum_{i=1}^{n_{o,cr}} P_{d,cr,i}(t)}{N_{tot}}$$ **Equation 10**

where $n_{o,cr}$ is the number of elements which have cracks larger than the detectable size, N_{tot} is the total number of hull structural criticality, C_i is the consequence of the failure of that element, and $P_{d,cr,I}(t)$ was defined above.

For corrosion, the analogous equations are

$$P_{d,cor,i}(t) = 1 - \exp\left(-\frac{ds_i \ln(1-p)}{ds_p}\right)$$ **Equation 11**

where $P_{d,cor,i}(t)$ is the probability of detecting corrosion within a specific structural element, ds_i is the fractional thickness affected, and ds_p is the thickness that can be detected with probability p.

For the entire hull, the average probability of detection of cracked elements is given by

$$P_{d,cor} = \frac{\sum_{i=1}^{n_{o,cor}} P_{d,cor,i}(t)}{N_{tot}}$$ **Equation 12**

where $n_{o,cor}$ is the number of elements with corrosion, N_{tot} is the total number of hull structural elements, and $P_{d,cor,i}(t)$ was defined above. As was the case above, the likelihood of detection is a function of the accessibility of the locations and the methods used for detection of corrosion. The quantities p and ds_p will therefore be parameters within the model.

The extent to which hull reliability renewal occurs is the extent to which the damage from cracking and corrosion is detected and the fraction of such damage that is repaired. That is

$$P_r(T_j) = P_{r,cr}(T_j)[1 - P_{r,cor}(T_j)] + [1 - P_{r,cr}(T_j)]P_{r,cor}(T_j) + P_{r,cr}(T_j)P_{r,cor}(T_j)]$$ **Equation 13**

where the P_r (Tj) is the probability of repair and restoration at time Tj. It is related to the probabilities of detections defined by eqs. 7 through 10 by x_{cr}, the fraction of detected cracks that are repaired, and x_{cor}, the fraction of detected corrosion damage that is repaired, i.e.,

$$P_{r,cr} = x_{cr} P_{d,cr} \text{ and } P_{r,cor} = x_{cor} P_{d,cor}$$

Equation 14

4. RESULTS OF USING THE MODEL

SoHaR has developed a prototype risk-based maintenance strategy tool (RBMST) was developed using the probabilistic model described in section 3 and the cost model described in section 2. The software calculates the reliability of individual structural details in terms of the corrosion and cracking degradation defined above and the stress/strength reliability prediction and the inspection effectiveness which is in turn affected by NDE. When the probability calculations are complete, the tool uses the maintenance and cost data from the to define risk (probability of loss multiplied by the economic consequences of the loss) and maintenance cost data to define the cost tradeoffs. These details are then integrated into intermediate structural elements (compartments) and then into the full ship model in accordance with the overall flow shown in Figure 2.

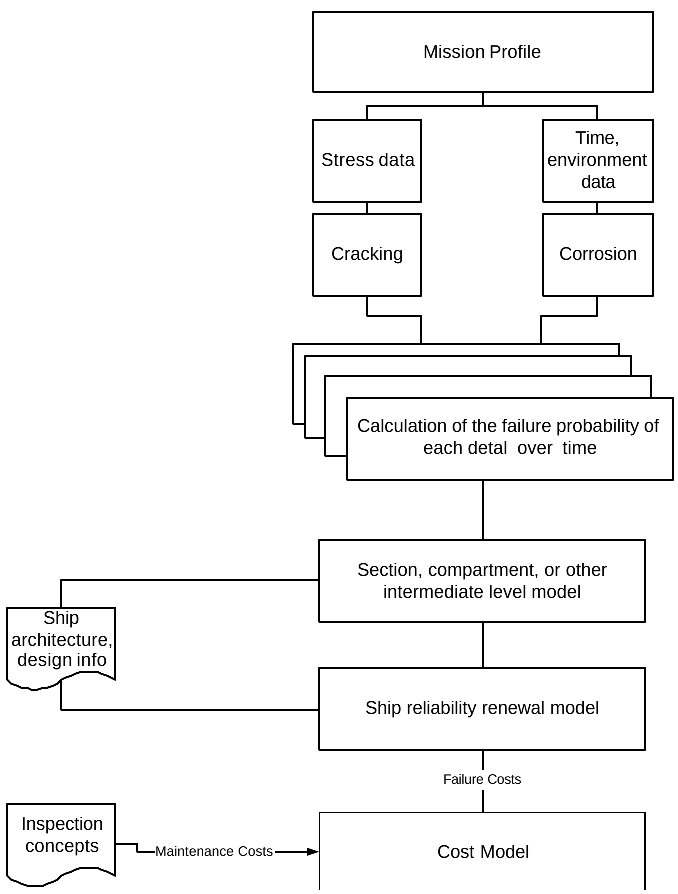

Figure 2. Overall calculational flow for RBMST

The remainder of this section discusses the results using the prototype with the following structural assumptions:

1. Ship sections are homogeneous
2. There are 3 detail types that make up respectively 30%, 50%, 20% of each section
3. The vessel consists of 12 compartments, 9 of which are required for the vessel to remain afloat.

It must be emphasized that these results, as well as the remaining results discussed in this section, are based on preliminary assumptions on the joint characteristics, distributions, and operational profiles. Two alternative maintenance strategies (strategies are a combination of inspection techniques used for each inspection) were considered

- Strategy 1: 65% effectiveness (primarily visual)
- Strategy 2: 85% effectiveness (includes NDE devices)

Figure 3 shows the total probability of ship survival calculated by RBMST using Strategy 1 and 5-year maintenance intervals. The results are presented in terms of a *reliability index*, an accepted measure of risk in which the probability of failure is expressed as a normalized variable. Table 3 shows the relationship between the reliability index and reliability The figure shows two calculations: with inspection (blue line) and without inspection (green line).

Table 2. Reliability Index and Probability of success (reliability)

Reliability Index	Probability of success (reliability)
0.5	0.691462
1	0.841345
1.5	0.933193
2	0.97725
2.5	0.99379
3	0.99865
3.5	0.999767
4	0.999968

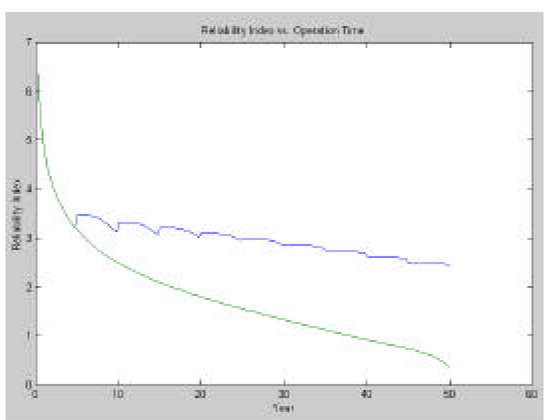

Figure 3. Ship failure probability using 5-year inspection intervals with a 65% inspection effectiveness (blue line) and no inspections (green line).

In both cases, as the ship ages, its reliability (as measured by its reliability index) decreases. However, the graph does show the benefits of periodic inspections. Whereas a ship inspected at 5-year intervals has a 99.87% reliability at the end of 50 years (reliability index of 3), the ship with no inspections has a reliability of less than 50%. The sawtooth shape of the with-inspection (blue) reliability curve is because of the reliability increase at the end of an inspection interval and demonstrates the"renewal" capability of our tool, i.e., that after an inspection, the ship is more reliable than it was before going in for inspection (and repair of any detected cracks or corroded areas).

Figure 4 shows the analogous case for a ship using Strategy 2, in which the inspection, due to addition of NDE effectiveness is 80%.

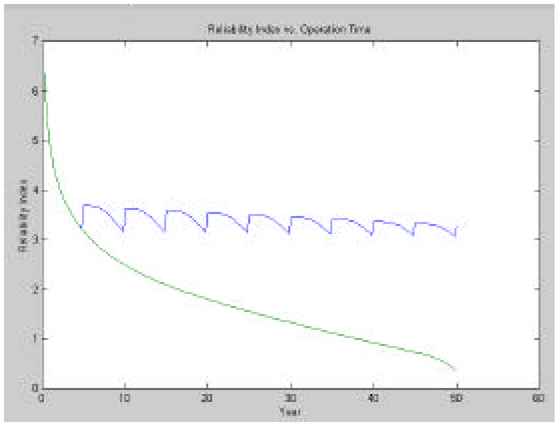

Figure 4. Ship failure probability using 5-year inspection intervals with an 80% inspection effectiveness due to addition of NDE (blue line) and no inspections (green line)

The results show the benefit of more effective inspections in terms of a higher reliability index (3.5 vs. 3), corresponding to a ship reliability of 99.98%. As will be discussed below, the increased cost of inspections can be measured against the decreased failure probability of the ship (0.13% for inspection strategy 1 vs. 0.02% for inspection strategy 2) to determine the economic benefit.

Another tradeoff that can be performed by RBMST is inspection interval. Figure 5 shows the result of 10-year inspection intervals with inspection strategy 1 (65% effectiveness). The average reliability index over this period is approximately 2.5, corresponding to a ship reliability of 99.4% and a failure probability of 0.6%. This result can be compares with a failure probability of 0.13% for a 5-year inspection interval (see Figure 3). As was the case for an increased inspection effectiveness, this result can be evaluated for cost-effectiveness in determining the maintenance strategy.

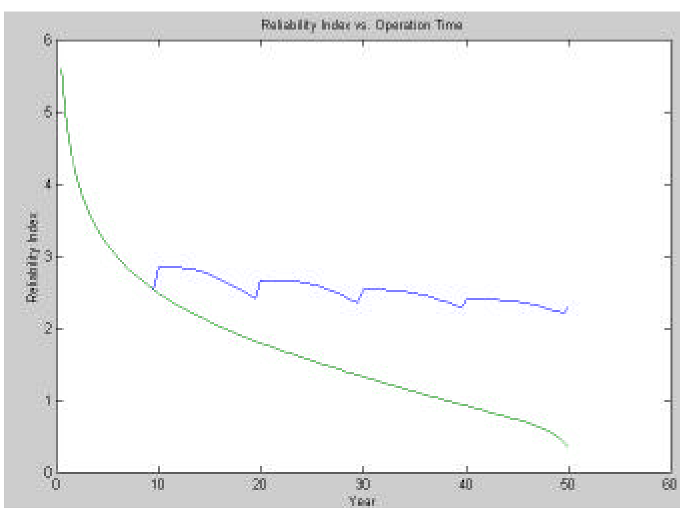

Figure 5. Ship failure probability using 10-year inspection intervals with an 65% inspection effectiveness (blue line) and no inspections (green line)

The overall goal of the RBMST is to develop cost-effective maintenance strategies. Equation 1 in section 2 showed the general relation used in the RBMST model which consisted of two terms: (1) a lifecycle cost term for inspection and repairs, and (2) a risk term to represent the consequence of a loss. The economic model was applied against the two maintenance strategies and inspection intervals describe above. Table 4 shows the results, table 5 shows the assumptions, and Figure 13 depicts the significant results in graphical terms

Table 3. Economic Model Results

Option	Inspection Strategy	Inspection Interval	Minimum reliability index	Maximum reliability index	Average failure probability	Maintenance Lifecycle Cost	Failure Risk Cost	Total cost
A	1	5	2.8	3.4	0.000967671	$1,865,097	$483,835	$2,348,933
B	1	10	2.6	2.9	0.002979819	$1,150,640.	$1,489,909	$2,640,549
C	2	5	3.1	3.8	0.000280341	$3,730,194.	$140,170	$3,870,365

Table 4. Assumptions used in economic model

Parameter	Assumed Value
Discount rate (for PV calculation)	5%
Cost of ship loss	$500,000,000
Inspection cost, strategy 1	$75,000 per inspection
Inspection cost, strategy 2	$150,000 per inspection

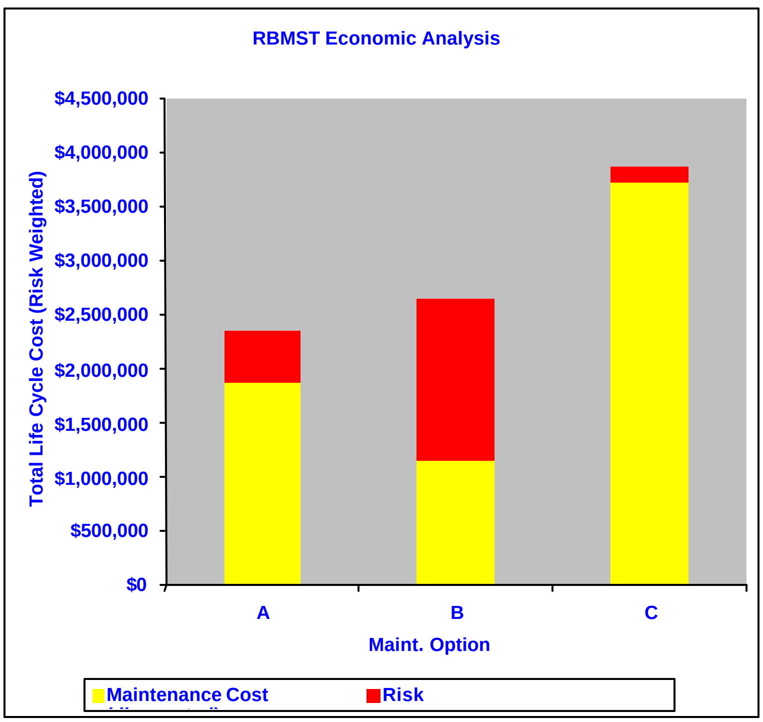

Figure 6 . Total Cost Results (sum of risk and maintenance related costs)

These results demonstrate that, under these assumptions, the lowest lifecycle cost result is achieved by using option A, i.e., using frequent moderate cost inspections and accepting a moderate level of risk. Option C, using high inspection intervals and a high cost method, results in the lowest risk but at a relatively high cost. If, however, the cost of a loss is higher, then the higher inspection strategy approach would be justified.

5. CONCLUSIONS

By means of integrating stress-strength probabilities at the structural detail level, system-level probabilistic models, and economics, the RBMST provides a framework for cost justifying the addition of NDE measures in ships. The same approach can be used for other civil structures.

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